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THE PROBLEM OF THE PASTURAGE. (SEE MATHEMATICAL MONTHLY, VOL. II, PAGE 82.)

BY ALEXANDER EVANS, ELKTON, MARYLAND.

THE 137th, and last of the Miscellaneous questions, in the 3rd part of Emerson's North American Arithmetic, which was published in 1835, is as follows:

"If 12 oxen eat up $3\frac{1}{2}$ acres of grass in 4 weeks, and 21 oxen eat up 10 acres in 9 weeks, how many oxen will eat up 24 acres in 18 weeks; the grass being at first equal on every acre, and growing uniformly?"

In June, 1835, a premium of $50 was offered for the most lucid analytical solution of this problem. A Committee was appointed, which reported 48 answers out of 112 to be correct; and by its chairman P. Mackintosh, awarded the prize to James Robinson, whose answer was $37\frac{113}{175}$. In 1860, a review of Mr. Robinson's solution was presented to the National Teacher's Association at Washington, by the Hon. Finley Bigger, then Register of the Treasury, with three solutions; $37\frac{113}{175}$, $37\frac{167}{175}$ and $21\frac{26}{35}$.

This review was printed in the 2nd Vol. of the *Mathematical Monthly*, **p.** 82; and was supplemented by an Editorial Note at page 85, containing an excellent solution, with the result $37\frac{113}{175}$.

But neither Mr. Emerson, nor the Committee, nor Mr. Robinson, nor Mr. Bigger, nor the National Teachers' Association alludes, nor do the Editors of the *Mathematical Monthly* allude to the fact that the question, with the exception of a curious misprint, is taken from the *Arithmetica Universalis* of Newton, which contains besides, a "lucid analytical solution", as the following extract from *Ralphson's* translation in the Edition of 1728 will show. — The Solutions are *Newton's*; as is the Example.

"*Arithmetical Questions*:" page 79, Problem XI.

"If the number of oxen a eat up the meadow b in the time c; and the number of oxen d eat up as good a piece of pasture e in the time f, and the grass grows uniformly; to find how many oxen will eat up the like pasture g in the time h.

"If the oxen a in the time c eat up the pasture b; then by proportion, the oxen $\frac{e}{b}a$ in the same time c, or the oxen $\frac{ec}{bf}a$ in the time f, or the oxen $\frac{ec}{bh}e$ in the time h will eat up the pasture e; supposing the grass did not grow at all after the time c.

"But since by reason of the growth of the grass, all the oxen d in the time f can eat up only the meadow e, therefore that growth of the grass in the meadow e in the time $f - c$ will be so much as alone would be sufficient to feed the oxen $d - \frac{eca}{bf}$ the time f, that is as much as would suffice to feed the oxen $\frac{df}{h} - \frac{eca}{bh}$ in the time h.

"And in the time $h - c$, by proportion so much would be the growth of the grass as would be sufficient to feed the oxen $\frac{h - c}{f - c}$ into $\frac{df}{h} - \frac{eca}{bh}$ or

$$\frac{bdfh - ecah - bdcf + aecc}{bfh - bch}.$$

Add this increment to the oxen $\frac{aec}{bh}$, and there will come out

$$\frac{bdfh - ecah - bdcf + ecfa}{bfh - bch}$$

the number of oxen which the pasture e will suffice to feed in the time h.

"And so in proportion the meadow g will suffice to feed the oxen

$$\frac{bdfgh - ecagh - bdcgf + ecfga}{befh - bceh}$$

during the same time h.

"EXAMPLE. If 12 oxen eat up $3\frac{1}{3}$ acres of pasture in 4 weeks, and 21 oxen eat up 10 acres of like pasture in 9 weeks; to find how many oxen will eat up 24 acres in 18 weeks? Answer 36: for that number will be found by substituting in

$$\frac{bdfgh - ecagh - bdcgf + ecfga}{befh - bceh}$$

the numbers 12, $3\frac{1}{3}$, 4, 21, 10, 9, 24, and 18, for the letters a, b, c, d, e, f, g, and h respectively; but the solution perhaps will be no less expedite, if it be brought out from the first principles, in form of the precedent literal solution.

"As if 12 oxen in 4 weeks eat up $3\frac{1}{3}$ acres, then by proportion 36 oxen in 4 weeks, or 16 oxen in 9 weeks, or 8 oxen in 18 weeks, will eat up 10 acres, on supposition that the grass did not grow.

"But since by reason of the growth of the grass 21 oxen in 9 weeks can eat up only 10 acres, that growth of the grass in 10 acres for the last 5 weeks will be as much as would be sufficient to feed the excess of 21 oxen above 16, that is 5 oxen for 9 weeks, or what is the same thing, to feed $\frac{5}{2}$ oxen for 18 weeks.

"And in 14 weeks (the excess of 18 above the first 4) the increase of the grass by analogy, will be such, as to be sufficient to feed 7 oxen for 18 weeks; for it is 5 weeks : 14 weeks :: $\frac{5}{2}$ oxen : 7 oxen, "Wherefore add these 7 oxen, which the growth of the grass alone would suffice to feed, to the 8, which the grass without growth after 4 weeks would feed, and the sum will be 15 oxen.

"And, lastly, if 10 acres suffice to feed 15 oxen for 18 weeks, then, in proportion, 24 acres would suffice 36 oxen for the same time." So far *Newton.*

Thus, it will be perceived that *Newton* had given a "lucid analytical solution" of this problem, about a century and a half before the prize of $50 was awarded to Mr. Robinson: it will also be seen that *Newton*'s example, and that given by Mr. Emerson, have the same identical quantities and figures, except, that the latter has by some means, purposed or accidental, substituted in his copy of *Newton's* example, $3\frac{1}{2}$ acres for $3\frac{1}{3}$ acres, whereby the somewhat ludicrous result is reached in the solution, that $\frac{113}{175}$ of an ox is at pasture for 18 weeks. *Newton's* data, as will be seen, give a whole number of oxen; to wit, 36 oxen.

We do not say that a player must make 24.6 throws with two dice, in order that he may undertake, on an equality of chance, to throw aces once; the $\frac{6}{10}$ of a throw being as impossible as the grazing of the $\frac{113}{175}$ of an ox; but we say that the probability of throwing aces once in 24 throws with two dice is less than $\frac{1}{2}$, and the probability of throwing aces once in 25 throws is more than $\frac{1}{2}$.

The 1st Edition of the Universal Arithmetic of *Newton* was published by *Whiston* in 1707, without *Newton's* consent, and it is said contrary to his wishes: it appeared at Cambridge, where the original was used in instruction after 1669; or during the time of Newton's Lucasian Professorship. It is proper to state that Whiston declared he had Newton's permission to publish.

The 2nd Edition; with Emendations said to be by *Newton,* was *Machin's* in 1722; at London in Latin.

An Edition, called the 2d: at London; translated by Ralphson, revised and corrected by Mr. Cunn: 1728. This is announced as "very much corrected" and as "carefully compared" with the Edition of 1722 which it calls "the correct Edition of the original." There was also an Edition of 1720, published at London; being, it is supposed, the first Edition of the preceding, or the second of Whiston.

The Leyden Edition of 1732 by S' Gravesande.

Aritmetica Universales, Newtoni. Quarto. 2vols. Amsterdam. 1761. cum Commentariis. Joh. Castillonei. This is said to be the best Edition. De Morgan. Arihmetical Books, page 73. However, Mr. Wilder, in the Edition next mentioned, says: "As to Castillioneus, I dislike chiefly three things in his book. First beside the great errors of the press, he is unnecessarily prolix. Secondly, he does not pay proper regard to his author's notation. Lastly the price and bulk of this book are too great in respect of its utility." *Brunet* says this Edition of 1760.

The Rev. Theaker Wilder's Edition; translated by Ralphson, revised and corrected by Cunn; to which is added a treatise upon the measure of ratios by James Maguire. London, 1769. Wilder succeeded Maguire as Professor of Mathematics at Trinity College, Dublin: took the pecuniary risk of the publication, and gave the profits to Maguire's representatives.

Editio, commentariis illustrata et aucta a J. A. Lecchi. Milan, 1752, 8vo. 3 vols.

Arithmetique Universelle, traduit du Latin, avec des notes par Noel Beaudeux. Paris, 1802: 4to; 2 vols.

Mr. Wilder mentions, Reyneau, Bernouilli, Maclaurin, Coleson and Campbell as among those persons who had illustrated some particular parts of the work.

The Universal Arithmetic deserves an American Edition, with notes and comments by some mathematician of ability.

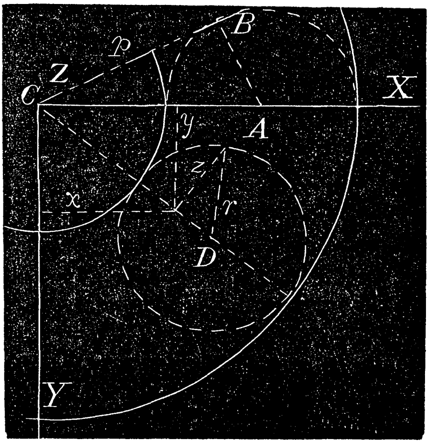
THE SECTION OF A CIRCULAR TORUS BY A PLANE PASSING THROUGH THE CENTER AND TANGET AT OPPOSITE SIDES.

BY PROF. E. W. HYDE, CIN. UNIV., CINCINNATI, OHIO.

In the diagram suppose the axes of x and y to be in the plane of the paper, and the axis of z perpendicular to the same.

For any point of the surface we shall have $x^2 + y^2 = \rho^2$, and $\rho = R - \sqrt{(r^2 - z^2)}$, in which $R = AC$ and $r = AB$.

$\therefore\ x^2 + y^2 = R^2 - 2R\sqrt{(r^2 - z^2)} + r^2 - z^2$, whence by transposing, squaring, reducing and placing $R^2 - r^2 = p^2$ and $R^2 + r^2 = q^2$ we obtain $(x^2 + y^2 + z^2)^2 - 2q^2(x^2 + y^2) + 2p^2z^2 = -p^4$.



The equation of a plane through the axis of y and tangent on the opposite sides above and below, is

$$z = \frac{rx}{\sqrt{(R^2 - r^2}} = \frac{r}{p}x.$$